

TA5 Project 5.3



Accelerated Testing and Preventive Maintenance in Acquisition, Maintenance and Operation of Vehicle Systems using TimeDependent Reliability / Durability Principles

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Army Needs in Reliability, Maintenance and Logistics

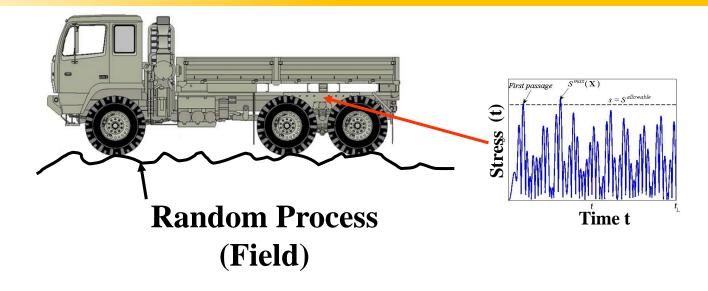


- > Reduce operations and maintenance costs
- Increase effectiveness of fleet logistics
- Control lifecycle cost and also use it in design and procurement
- Improve availability; schedule maintenance
- Use both analytical and experimental / field data to estimate reliability



Background







Random Process leads to Time-Dependent Reliability



Research Statement



Develop methodologies to obtain a preventive maintenance schedule and to assess and improve the reliability / durability of vehicle systems using

- Experimental (field) data
- "Expert" opinion

Previously and currently at TARDEC

Predictive tools (physics-of-failure data)

Current research



Overview



Part 1:

Optimal preventive maintenance schedule using time-dependent reliability and lifecycle cost

Part 2:

Accelerated testing method based on importance sampling using few tests which run for only a short time





Part 1: Optimal Preventive Maintenance Schedule



What is Reliability? **Cumulative Probability of Failure**



Reliability at time t is the probability that the system has not failed before time t.

$$F_T^c(t_L) = P(\exists t \in [0, t_L], such that $g(\mathbf{X}(t), t) \leq 0)$ Cumulative Prob. of Failure$$

$$F_T^i(t_L) = P(g(\mathbf{X}(t_L), t_L) \le 0)$$

 $F_T^i(t_L) = P(g(\mathbf{X}(t_L), t_L) \le 0)$ Instantaneous Prob. of Failure

Calculation Methods for $F_{\tau}^{c}(t)$

- Maximum Response Method
- Niching GA & Lazy Learning Local Metamodeling

• MCS / Importance sampling
$$F_T^c(t) = 1 - \exp\left[-\int_0^t \lambda(t)dt\right]$$

Simulation-based



Definition of Lifecycle Cost



Lifecycle Cost = Production Cost

+Inspection Cost

Expected Variable Cost

Quality

Time-Dependent System Reliability



Definition of Lifecycle Cost



$$C_{L}(\mathbf{d}, \mathbf{X}, t_{f}, r) = C_{P}(\mathbf{d}, \mathbf{X}) + C_{I}(\mathbf{d}, \mathbf{X}, t_{0}) + C_{V}^{E}(\mathbf{d}, \mathbf{X}, t_{f}, r)$$
Lifecycle
Production
Inspection
Cost
Cost
Cost
Variable Cost

Final time Interest rate
$$C_V^E(\mathbf{d}, \mathbf{X}, t_f, r) = \int_0^{t_f} c_F(t) e^{-rt} f_T^c(t) dt$$
Cost of failure PDF of time to failure time

$$F_T^c(t_L) = P(\exists t \in [0, t_L], such that g(\mathbf{X}(t), t) \leq 0)$$



Preventive Maintenance Schedule



Estimation of Time for Preventive Maintenance

$$\max_{\mathbf{d}, \mathbf{\mu}_{\mathbf{X}}, \mathbf{\sigma}_{\mathbf{X}}, t_M} t_M$$

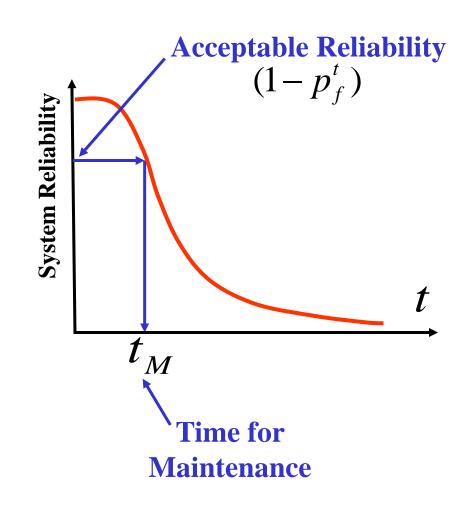
s. t.
$$C_L(\mathbf{d}, \boldsymbol{\mu}_{\mathbf{X}}, \boldsymbol{\sigma}_{\mathbf{X}}, t_M, r) \leq C_L^t$$

$$F_T^c(\mathbf{d}, \mathbf{X}, t_M) \leq 1 - R^t(t_M)$$

$$\mathbf{d}_{I} \leq \mathbf{d} \leq \mathbf{d}_{II}$$

$$\mu_{X_{\mathit{L}}} \leq \mu_{X} \leq \mu_{X_{\mathit{U}}}$$

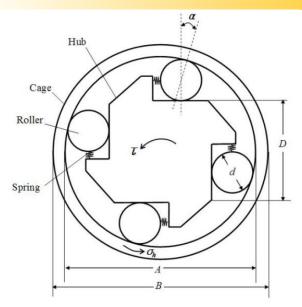
$$\sigma_{\mathbf{X}_{L}} \leq \sigma_{\mathbf{X}} \leq \sigma_{\mathbf{X}_{U}}$$





A Roller Clutch Example





Random Variables: D, d, A

Due to degradation:

$$\mathbf{D} \to \mathbf{D}(1-kt)$$

$$\mathbf{d} \rightarrow \mathbf{d} (1 - kt)$$

$$\mathbf{A} \to \mathbf{A}(1+kt)$$

with: $k = 2.5E - 04 \, mm / year$

Constraints:

 \rightarrow Contact angle $\alpha = 0.11 \pm 0.06$ rad

 \rightarrow Torque $\tau >= 3000$ Nm

 \longrightarrow Hoop stress $\sigma_h \le 400$ MPa

$$g_1(D,d,A) = 0.05 - \cos^{-1}\left(\frac{D-d}{A-d}\right) \le 0$$

$$g_2(D,d,A) = \cos^{-1}\left(\frac{D-d}{A-d}\right) - 0.17 \le 0$$

$$g_3(D,d,A) = 3000 - NL \left(\frac{\sigma_c}{c_1}\right)^2 \frac{D^2 d}{4(D+d)} \sqrt{1-S^2} \le 0$$

$$g_4(D,d,A) = \frac{N}{2\pi} \left(\frac{\sigma_c}{c_1}\right)^2 \left(\frac{Dd}{(D+d)}\right) \frac{S}{A} \left(\frac{B^2 + A^2}{B^2 - A^2}\right) - 400E06 \le 0$$



Roller Clutch: Lifecycle Cost



$$C_L = C_P + C_I + C_V^E$$

where:

$$C_P = \left(3.5 + \frac{0.75}{3\sigma_D}\right) + \left(3.0 + \frac{0.65}{3\sigma_d}\right) + \left(0.5 + \frac{0.88}{3\sigma_A}\right)$$

$$C_I = 20F_T^i(\mathbf{X}, t_0)$$
Scrap cost/unit

$$C_{I} = 20F_{T}^{i}(\mathbf{X}, t_{0})$$

$$C_{V}^{E} = \int_{0}^{t_{f}} 20e^{-rt} f_{T}^{c}(t) dt$$

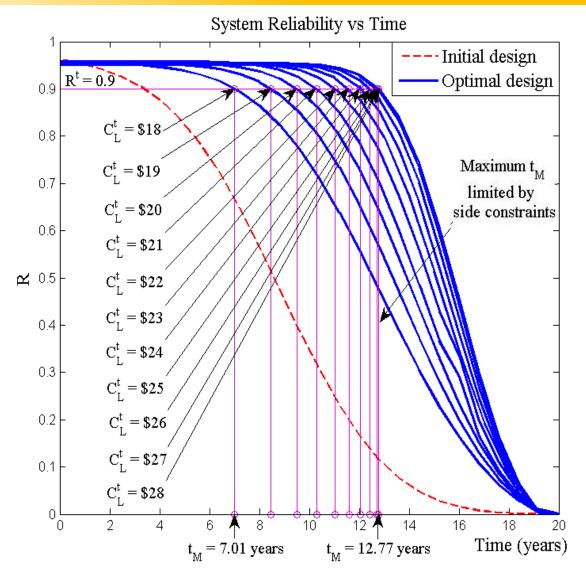
$$t_{f} = 10 \quad years$$
Failure cost/unit (warranty cost)

Failure cost/unit (warranty cost)



Roller Clutch: Reliability vs Time-to-Maintenance

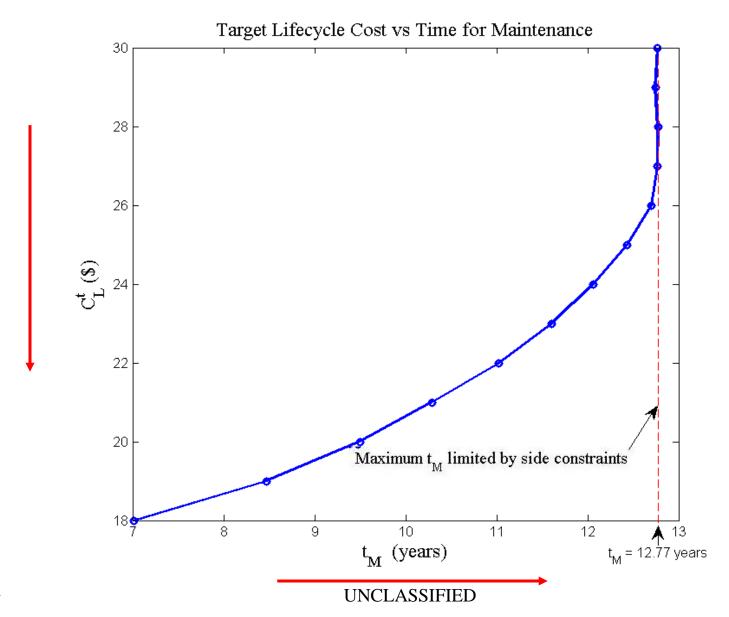






Roller Clutch: Pareto Optimality between Time-to-Maintenance and Cost







Roller Clutch: Pareto Optimality between Time-to-Maintenance and Cost



Design Variables:

$$\boldsymbol{\mu}_{\mathbf{X}} = \left\{ \mu_D, \, \mu_d, \, \mu_A \right\} \qquad \boldsymbol{\sigma}_{\mathbf{X}} = \left\{ \boldsymbol{\sigma}_D, \, \boldsymbol{\sigma}_d, \, \boldsymbol{\sigma}_A \right\}$$

Side Constraints:

$$55.0973 \le \mu_D \le 55.4973$$

$$22.66 \le \mu_D \le 23.06$$

$$101.49 \le \mu_A \le 101.89$$

$$0.04 \le \sigma_D \le 0.08$$

$$0.03 \le \sigma_d \le 0.1$$

$$0.07 \le \sigma_A \le 0.113$$

I											
c_L^t	18	19	20	21	22	23	24	25	26	27	28
$\mu_{\scriptscriptstyle D}$	55.4946	55.4973	55.4973	55.3822	55.4973	55.4973	55.4973	55.4973	55.4973	55.4973	55.4973
μ_d	22.7562	22.7735	22.7867	22.8535	22.8071	22.8146	22.8208	22.8259	22.8296	22.8315	22.8316
μ_{A}	101.49	101.49	101.49	101.49	101.49	101.49	101.49	101.49	101.49	101.49	101.49
$\sigma_{\scriptscriptstyle D}$	0.08	0.08	0.0771	0.0693	0.0661	0.0593	0.054	0.0496	0.0423	0.04	0.04
σ_d	0.0639	0.0543	0.0481	0.0449	0.0407	0.0368	0.0334	0.0306	0.03	0.03	0.03
$\sigma_{_A}$	0.1107	0.0946	0.084	0.0763	0.0701	0.07	0.07	0.07	0.07	0.07	0.07





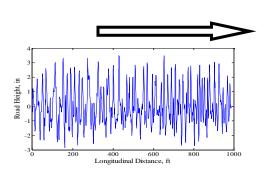
Part 2: Accelerated Testing using Importance Sampling



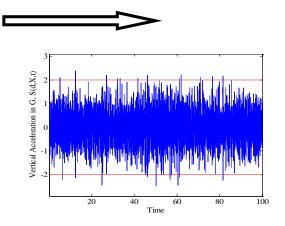
Problem Description







Vertical Accel. (G)



Terrain

Vehicle speed: 20 mph; Mission distance: 100 miles

Simulation can be practically performed for a short-duration time



Our Approach



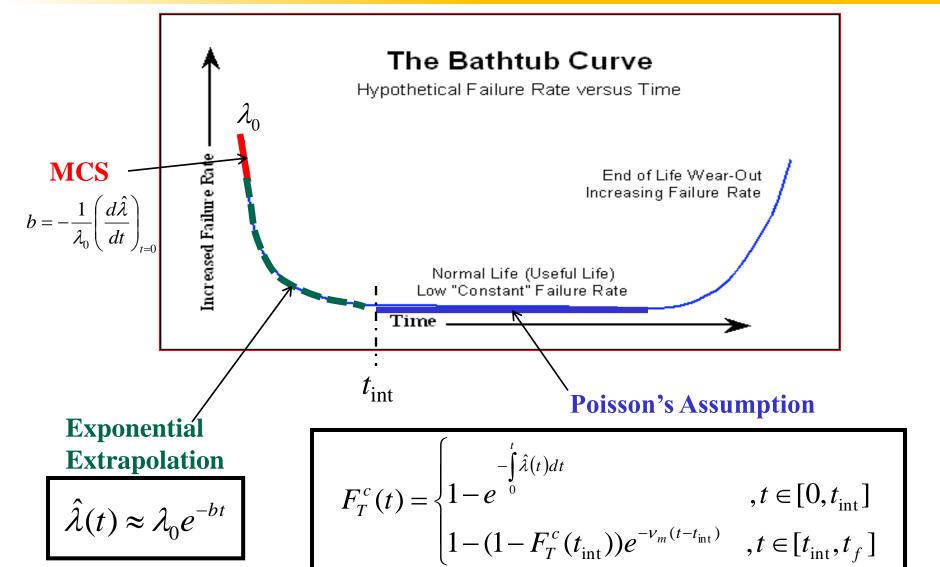
A novel MC-based method to calculate the timedependent reliability (cumulative probability of failure) based on:

- ➤ short-duration data and an exponential extrapolation using MCS or Importance Sampling (Infant Mortality)
- > Poisson's assumption (Useful Life)



Efficient MC Simulation Approach







Poisson Assumption



$$F_T^c(t_{\min},t) = 1 - (1 - F^i(t_{\min}))e^{-m_1}$$

where:

$$m_1 = E[N^+(t_{\min}, t)] = \int_{t_{\min}}^{t} v^+(t) dt = v_m(t - t_{\min})$$

Number of out-crossings

$$v^{+}(t) = \lim_{\Delta \tau \to 0, \Delta \tau > 0} \frac{P[g(\mathbf{d}, \mathbf{X}, t) > 0 \cap g(\mathbf{d}, \mathbf{X}, t + \Delta \tau) \le 0]}{\Delta \tau}$$

Out-crossing rate



Quarter-Car Model on Stochastic Terrain Oakland



Constant design parameters:

 $m_s = 1000 \text{ kg}$ $m_{\rm u} = 100 \text{ kg}$ Vehicle speed = 20 mph

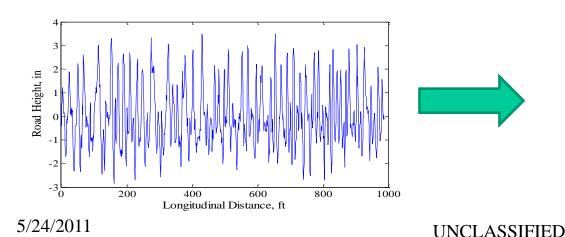


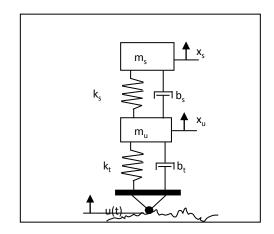
Random Input variables

Damping, $b_s \sim N(7000, 1400^2)$

Stiffness, $k_s \sim N(40 \times 10^3, (4 \times 10^3)^2)$

Random Input Process: Experimental Stochastic Terrain from Yuma Proving Grounds.

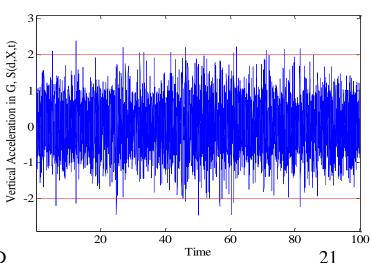




Random Output Process

(Vertical Acceleration, G')

Threshold = 2G



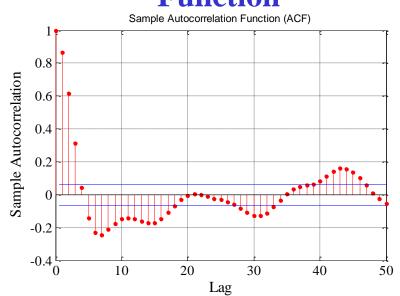


Quarter-Car Model: Road Input Random Process Characterization

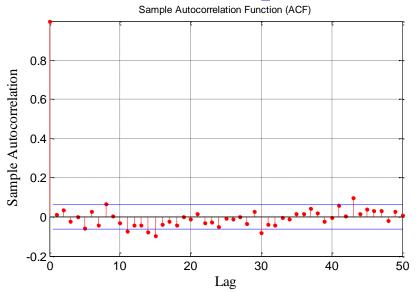


AR(3) model was identified based on:

Autocorrelation Function



Autocorrelation of Residual process



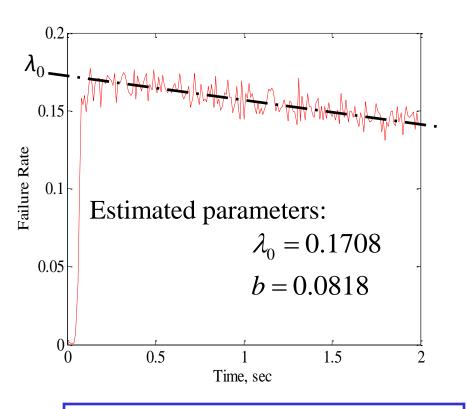
$$u_i = 1.2456$$
 $u_{i-1} - 0.2976$ $u_{i-2} - 0.1954$ $u_{i-3} + \varepsilon_i(0, 0.5132^2)$

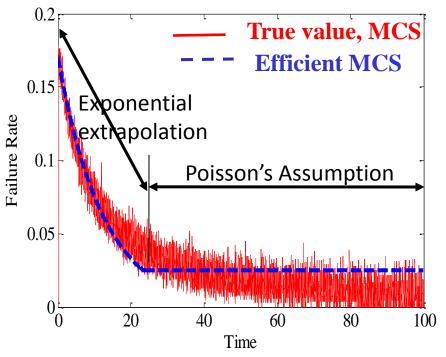
Statistical tests were performed to verify the model



Quarter-Car Model: Results(Failure Rate Estimation for Threshold = 2G)







Estimation requires short duration MCS

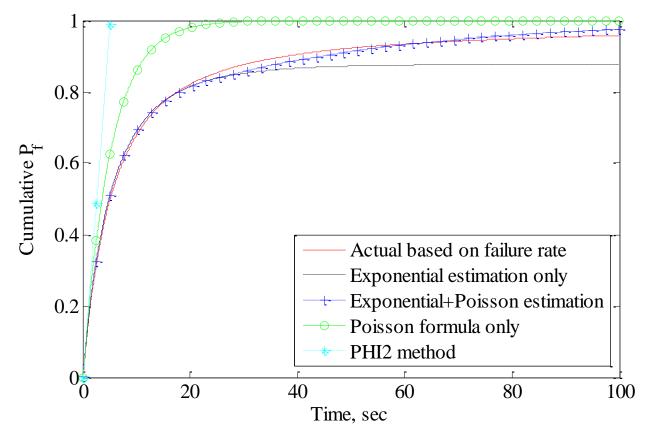
Exponential extrapolation

$$\hat{\lambda}(t) \approx \lambda_0 e^{-bt}$$



Quarter-Car Model: ResultsCumulative Probability of Failure for Threshold = 2G



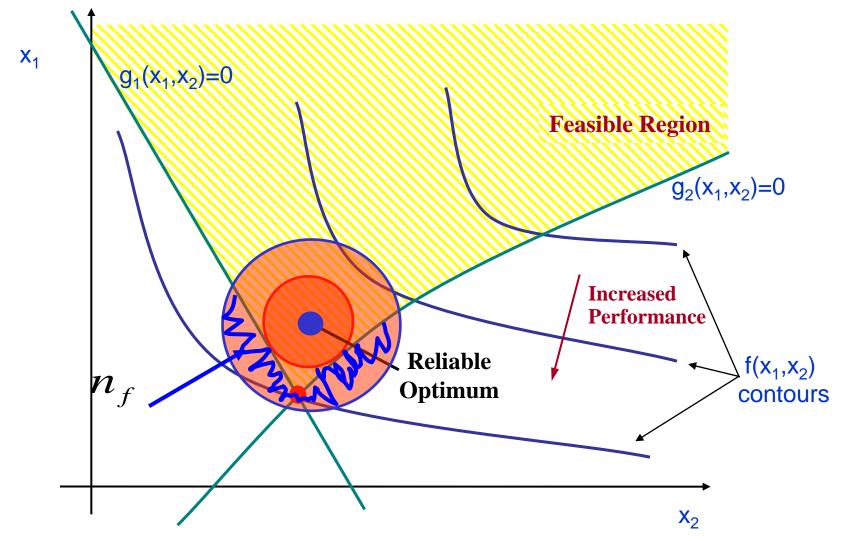


Efficient MCS (blue) approach is close to true MCS results (red)



Principle of Importance Sampling: Random Variable Case







MARC Importance Sampling for Random Process Oakland



Instantaneous Conditional Probability of Failure:

$$p_f^{\lambda}(t_i) = \int_{\Omega} \theta(\mathbf{x}; t_i) f_{\mathbf{X}}(\mathbf{x}; t_i) d\mathbf{x}$$

 $\mathbf{x} = \{x_1, x_2, \dots, x_i\}$ where x_i is a realization of R.V. $X_i = X(t_i)$

$$p_f^{\lambda}(t_i) = \int_{\Omega} \theta(\mathbf{x}; t_i) \frac{f_{\mathbf{X}}(\mathbf{x}; t_i)}{f_{\mathbf{X}_{-}^{S}}(\mathbf{x}; t_i)} f_{\mathbf{X}_{-}^{S}}(\mathbf{x}; t_i) d\mathbf{x}$$

Sampling Joint PDF

$$p_f^{\lambda}(t_i) = \frac{\sum_{n=1}^{N_s(t_{i-1})} \theta(\mathbf{x}; t_i) \omega(\mathbf{x}, t_i)}{N_S(t_{i-1})} = \frac{\sum_{n=1}^{N_f(t_i)} \omega(\mathbf{x}, t_i)}{N_S(t_{i-1})}$$



ARC Importance Sampling for Random Process Oakland



$$p_f^{\lambda}(t_i) = \frac{\sum_{n=1}^{N_s(t_{i-1})} \theta(\mathbf{x}; t_i) \omega(\mathbf{x}, t_i)}{N_S(t_{i-1})} = \frac{\sum_{n=1}^{N_f(t_i)} \omega(\mathbf{x}, t_i)}{N_S(t_{i-1})}$$

$$\lambda(t_i) = \lim_{\Delta t \to 0} \frac{p_f^{\lambda}(t_i)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\sum_{n=1}^{N_f(t_i)} \omega(\mathbf{x}, t_i)}{\Delta t \cdot N_S(t_{i-1})}$$

$$\omega(\mathbf{x}, t_i) = \frac{f_{\mathbf{X}}(\mathbf{x}; t_i)}{f_{\mathbf{X}^S}(\mathbf{x}; t_i)} : \text{Likelihood ratio at } t_i$$

 $N_{S}(t_{i-1})$: Safe sample points at t_{i-1}

 $N_f(t_i)$: Number of failures in $\Delta t = t_i - t_{i-1}$



Importance Sampling for Random Process Oaklan



Likelihood ratio:

$$\omega(\mathbf{x};t_i) = \frac{f_{\mathbf{X}}(\mathbf{x};t_i)}{f_{\mathbf{X}}^{S}(\mathbf{x};t_i)} = \frac{f_{\mathbf{X}}(x_i, x_{i-1}, \dots, x_{i-d})}{f_{\mathbf{X}}^{S}(x_i, x_{i-1}, \dots, x_{i-d})}$$

Decorrelation length: Maximum number of lags over which realizations of x_i are significantly correlated

$$x_i - \mu = \phi_1(x_{i-1} - \mu) + \phi_2(x_{i-2} - \mu) + \dots + \phi_p(x_{i-p} - \mu) + \varepsilon_i(N(0\sigma_s^2))$$

To generate sampling PDF

$$f_{\mathbf{X}}(\mathbf{x}) = \frac{1}{(2\pi)^{k/2} |\mathbf{\Sigma}|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{x} - \mathbf{\mu})^T \mathbf{\Sigma}^{-1} (\mathbf{x} - \mathbf{\mu})\right)$$

From Yule-Walker Eqs



ARC Importance Sampling for Random Process Oakland UNIVERSITY



Estimation of Safe Sample Functions

$$\lambda(t_i) = \lim_{\Delta t \to 0} \frac{\sum_{n=1}^{N_f(t_i)} \omega(\mathbf{x}, t_i)}{\Delta t \cdot N_S(t_{i-1})}$$

$$\frac{\sigma_e}{\sigma_S} x_f > S_{threshold}$$

"Inflated" response

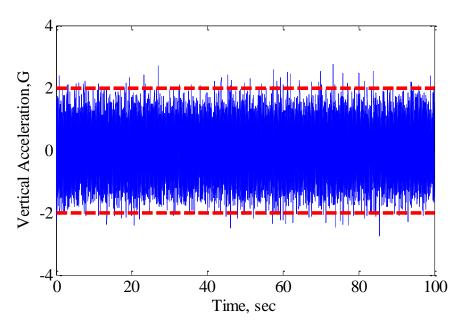


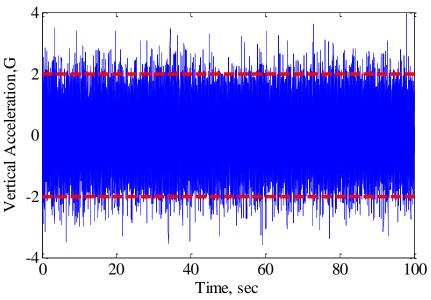


$$u_i = 1.2456u_{i-1} - 0.2976u_{i-2} - 0.1954u_{i-3} + \varepsilon_i(0, 0.5132^2)$$

Original PDF $\sigma_e = 0.51$

Sampling PDF $\sigma_s = 0.7$

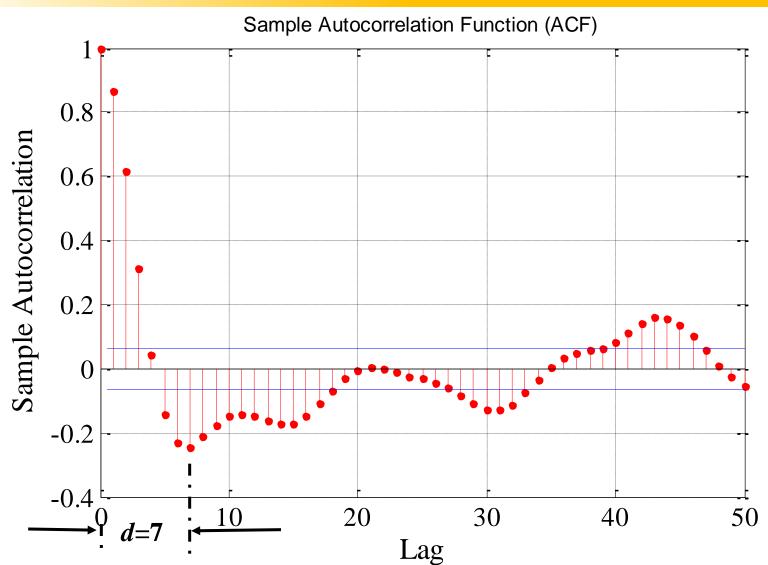




The sampling PDF results in more failures



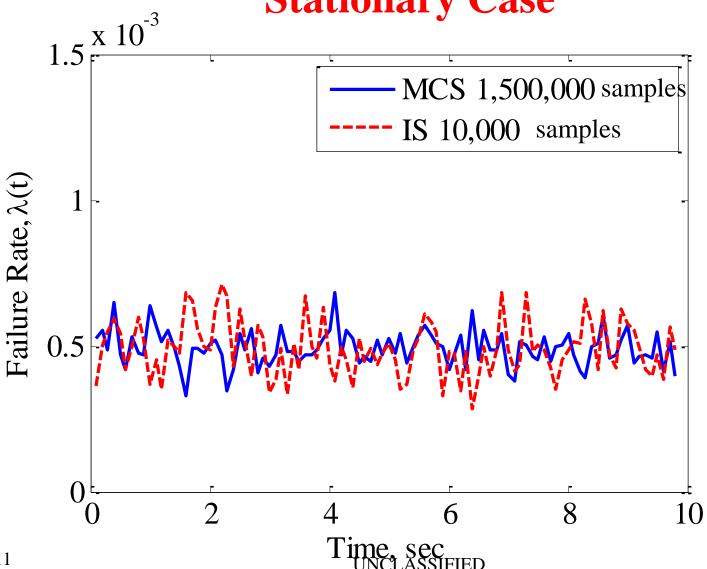








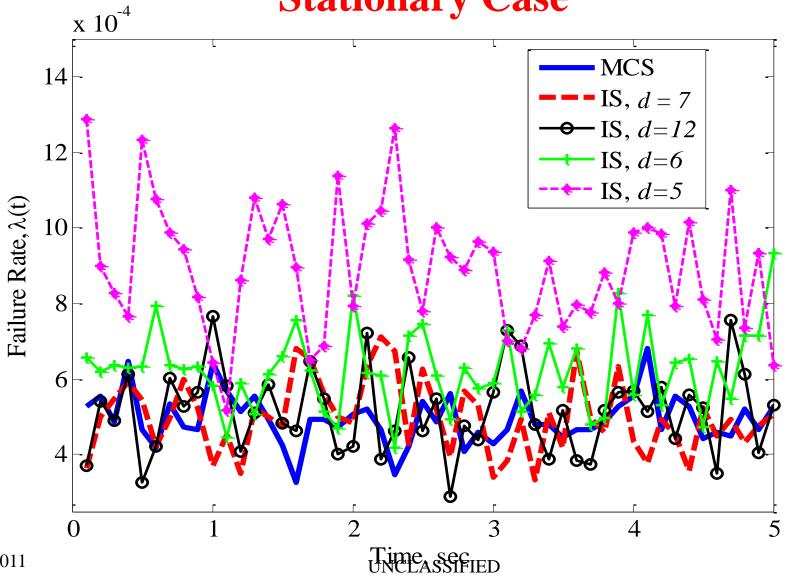










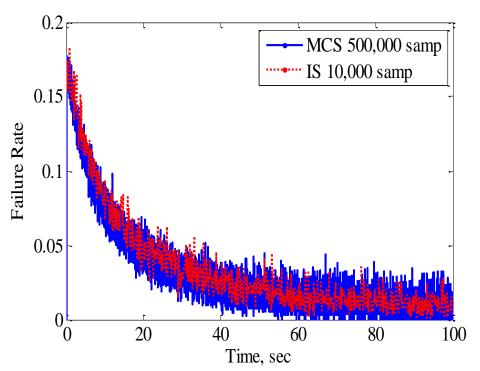




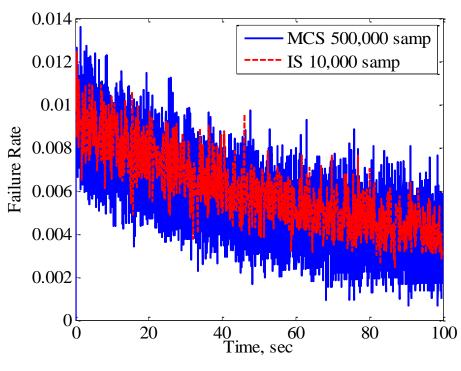


Non-Stationary Case





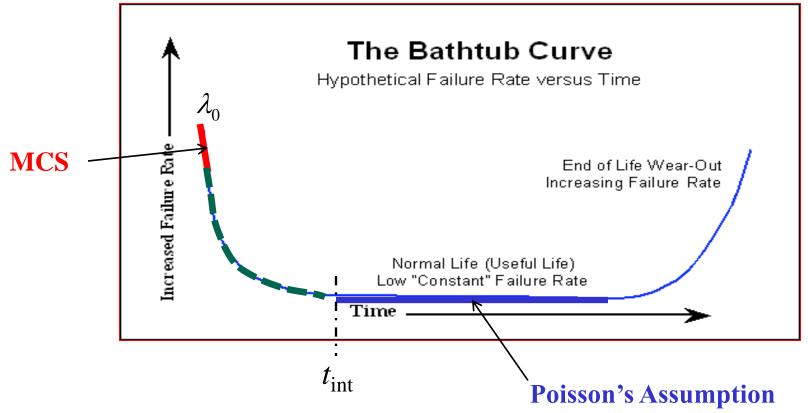
Threshold = 2.65 g





Observations / Practical Issues





- > Analytical methods can be used under the Poisson's assumption
- > IS at initial time may need a few thousand output sample functions



Ongoing Work Plan



- ➤ Improve the current accelerated testing method based on importance sampling so that only 5-10 tests are needed (Q3)
 - ✓ Characterize the "inflated" output random process in importance sampling using "generalized" Kriging and MLE and/or time series
- ➤ Demonstrate the accelerated testing methodology using the N-post (or 4-post) Reconfigurable Road Simulator of the Physical Simulation Laboratory at TARDEC (Q3 and Q4)



TARDEC N-post Reconfigurable Road Simulator









Thanks for your attention! Q&A

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